

Ephemerides of Variable Stars by David H. Bradstreet Ph.D. (Eastern University)

An ephemeris is a listing of the times when a characteristic of a changing object will take place, i.e., when will it be in a certain position, when will it have a certain brightness, when will an eclipse take place, etc. Ephemeral means always changing or not lasting for very long time. The ephemeris of a variable star is an equation that tells us when it will be at a certain brightness, usually deepest eclipse for an eclipsing binary. If the star is well behaved, the star's ephemeris will be a rather straightforward equation. Let us assume that the star is well behaved. We will now define some terms which we'll need for the ephemeris.

Period P = the time to make one complete orbit, usually expressed in days

Dates are usually expressed in **Julian Day** numbers to avoid the usual confusion with leap years and calendar changes. The Julian Day number system was invented in 1583 by Joseph Justus Scaliger, born August 5, 1540 in Agen, France, died January 21, 1609 in Leiden, Holland. The Julian Day numbering system began on January 1, 4713 BC and starts at noon in Greenwich so that observers at night will not have the day number change on them while they are observing. The conversion between regular date and JD can be done from tables in the Astronomical Almanac or from mathematical algorithms on a computer (see Meeus' book Astronomical Algorithms).

Epochal time of minimum light **JD₀** = time of minimum light in the light curve, usually the deepest eclipse (primary eclipse), as opposed to the other less deep eclipse (secondary eclipse).

To predict the time of minimum light (primary eclipse), one needs to know a previous time of minimum light (JD₀) and the binary's Keplerian (orbital) period. Suppose that JD₀ for VW Cep is 2450596.6586, and we know from previous workers that its period is 0.27831460 days. The next primary eclipse of VW Cep would occur exactly one orbital period later, i.e.,

$$JD(\text{next eclipse}) = JD_0 + 0.27831460(1)$$

The next eclipse would occur exactly two periods from JD₀, namely

$$JD(\text{2nd eclipse}) = JD_0 + 0.27831460(2)$$

And of course the third eclipse after T₀ would occur three periods later, i.e.,

$$JD(\text{3rd eclipse}) = JD_0 + 0.27831460(3)$$

We can therefore predict the time of any primary eclipse by specifying the number of periods past JD₀ that we wish. This number of periods is called the **epoch E**. So we have

$$JD(\text{primary eclipse}) = JD_0 + PE$$

If the eccentricity of the binary's orbit is zero (i.e., the orbit is a circle), then the secondary eclipse occurs exactly halfway (0.5) through the cycle. Thus we can predict secondary eclipses by simply specifying Epochs such as 0.5, 1.5, 2.5, etc. If the orbit is eccentric, then we will need to know the specific phase of the secondary eclipse.

Phase: Period Normalization

Typically observations take place over many nights and we will want to combine these nights into one complete light curve. To do this, we introduce the concept of phase, representing the time of one complete orbital period. Phase is defined from 0.0000 to 1.0000, where this would represent the total period of the binary. Primary eclipse is usually defined as phase 0.00P, secondary as 0.50P (assuming a circular orbit), running through one full period of 1.00P when we'd start over again, *i.e.*, 1.00P = 0.00P. We can calculate the phase of a particular observation using the binary's ephemeris:

$$JD(\text{observation}) = T_0 + PE$$

But this time we want to solve this equation for the Epoch:

$$E = \frac{JD(\text{obs.}) - T_0}{P}$$

The integer part of E represents the epoch of that particular observation, *i.e.*, how many full cycles (periods) have passed since T_0 . The decimal part of E is the phase of the observation, *i.e.*, at what fractional part of the next period did this particular observation occur.

For example, using the above ephemeris for VW Cep, let's calculate the epoch and phase for 9:00 PM EDT, August 26, 1997. First convert the EDT (daylight) to EST (standard) = 8:00 PM EST = 20:00 EST military time. Now convert to UT (Universal Time) by adding 5 hours (the difference in longitude in hours between us and Greenwich = 25:00 UT = 1:00 UT August 27. This means that 13 hours have past since the Julian Day began (since it begins at noon in Greenwich), and $13/24 = 0.5417$. Using the Julian Day algorithm we find that the JD for August 26, 1997 is 2450686, so the JD for our time is $2450686 + 0.5417 = 2450686.5417$. Calculating the epoch E and phase:

$$E = \frac{2450686.5417 - 2450596.6586}{0.27831460} = 322.9549$$

So, 322 complete orbits have transpired since our JD₀, (2450596.6586), and the fractional part of the number is the binary's phase at 9:00 PM EDT, namely 0.9549, soon to be primary eclipse.

To predict the phase of a binary from night to night (useful for filling in portions of a light curve you might be missing), calculate the phase for a given night and observation time:

$$\text{Phase} = \text{Fractional part}\left(\frac{JD(\text{obs.}) - JD_0}{P}\right)$$

We can then just keep adding 1 to the JD and calculate E again or, more simply, add the amount of phase that the system changes per day. To see how this works, consider a binary with a period $P = 0.90$ days. Let's assume that its ephemeris is given by:

$$JD = 2450592.8713 + 0.90E$$

We can calculate the phase on JD2450623.7000 as

$$\text{Phase} = \text{Fractional part}\left(\frac{2450623.7000 - 2450592.8713}{0.90}\right) = \text{Frac.}(34.2541) = 0.2541$$

So the phase of the star at 2450623.7000 is 0.2541P. What will it be exactly one day later, *i.e.*, 2450624.7000?

Calculating as before

$$\text{Phase} = \text{Fractional part}\left(\frac{2450624.7000 - 2450592.8713}{0.90}\right) = \text{Frac.}(35.3652) = 0.3652$$

Note that one day later we've gone through somewhat more than 1 period (1 epoch). In fact in one day we've gone through

$$35.3652 - 34.2541 = 1.1111 = \frac{1}{0.90} = \frac{1}{P}$$

So, the phase has increased by 0.1111 over one day since the period is slightly less than one day. Thus we could also have found the phase exactly one day later by simply adding 0.1111 to the first phase found for each successive day, i.e.,

$$0.2541 + 0.1111 = 0.3652$$

$$0.3652 + 0.1111 = 0.4763$$

$$0.4763 + 0.1111 = 0.5874$$

ad nauseum

What if the period is greater than 1 day? The star's phase will lag behind $(1 - \frac{1}{P})$ in phase. To demonstrate this, let's do our previous example again, but this time let the period of the binary be $P = 1.1$ days.

$$\text{Phase} = \text{Fractional part}\left(\frac{2450623.7000 - 2450592.8713}{1.1}\right) = \text{Frac.}(28.0261) = 0.0261$$

$$\text{Phase} = \text{Fractional part}\left(\frac{2450624.7000 - 2450592.8713}{1.1}\right) = \text{Frac.}(28.9352) = 0.9352$$

Note that since $P > 1$ day, we have not progressed one full cycle in the Epoch. The phase has increased by

$$0.9091 = \frac{1}{P} = \frac{1}{1.1}$$

So, to produce the next day's phase, we add this to the original phase (or subtract $(1 - \frac{1}{1.1}) = 0.0909$). But to reduce the chance of error, it is best to be consistent and

always add $\frac{1}{P}$.

Heliocentric light time correction

Although the speed of light is very fast, it is not infinite, and the incredible astronomical distances we are dealing with can lead to easily measurable timing effects due to light's finite speed. One such light time effect occurs because of the Earth's orbit around the Sun. Although the Earth's semimajor axis is only 8.3 light minutes in size, this moving platform of an Earth can therefore be several light minutes closer to or further away from stars, especially those near the ecliptic. To account for this variation, astronomers recalculate Julian Days as if we were observing the star from the center of the Sun, i.e., a Heliocentric Julian Day. This is usually calculated by computer, and its theory and formula derivation are detailed in Binnendijk (1960).

Analyzing Period Variations

The main analytical tool for studying period changes in binary stars is the so called **O-C diagram**. After collecting a number of eclipse timings, one can analyze how a binary's period is behaving by plotting observed (O) times of minimum light minus the predicted or calculated (C) times of minimum light (hence O-C) on the ordinate scale versus Epoch on the abscissa scale. If the estimated period P_{est} used for the predictions (C) is exactly equal to the actual period P , then the value of the O-C's would all be zero, and your plot would be a simple horizontal line. Of course, that almost never happens. Let us look a little more closely (mathematically) at how to understand an O-C diagram.

Derivation of O-C equations

O	=	time of observed minimum light
C	=	calculated time of minimum light
JD_0	=	time of minimum light at $E = 0$
P_{est}	=	estimated period
$P(E)$	=	true period of system, possibly a function of E (time)

We write expressions for the observed time of minimum light O and the calculated time of minimum light C:

$$O = JD_0 + P(E)E$$

$$C = JD_0 + P_{est}E$$

$$O - C = (P(E) - P_{est})E$$

Assume that the O-C residuals can be fit by a parabola:

$$O - C = aE^2 + bE + c$$

Equate the expressions for O-C:

$$(P(E) - P_{est})E = aE^2 + bE + c$$

$$P(E)E - P_{est}E = aE^2 + bE + c$$

Differentiate with respect to E (E is really time):

$$\frac{dP}{dE}E + (P(E) - P_{est}) = 2aE + b$$

Comparing terms on both sides of the equation we find that:

$$\frac{dP}{dE} = 2a$$

$$(P(E) - P_{est}) = b$$

If $P(E)$ is constant (i.e., not a function of E), we have:

$$\frac{dP}{dE} = 2a = 0$$

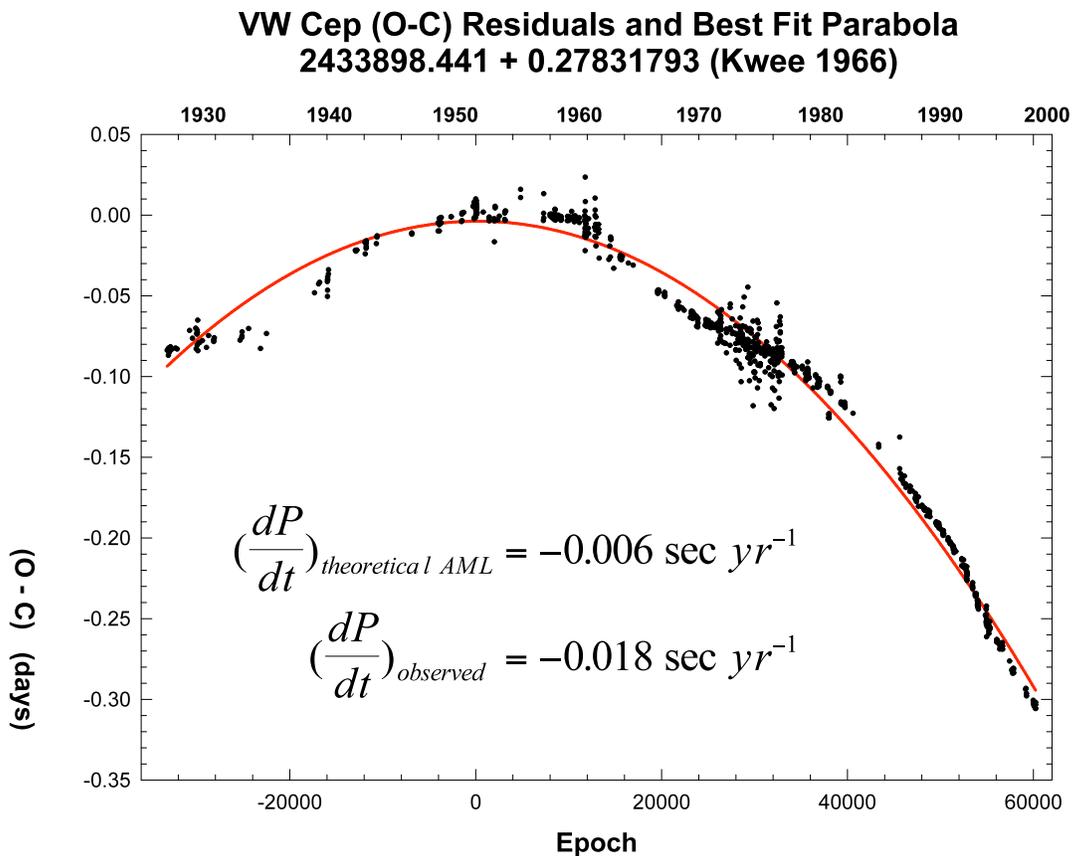
$$(P - P_{est}) = b$$

The O-C diagram would be a straight line since the quadratic term a is zero. The slope of the line (b) would be equal to the difference in the true period P and the estimated one P_{est} . Thus to correct P_{est} one would only have to add the value of the slope to it, i.e.:

$$P = P_{est} + b$$

Example: Fit to VW Cep O-C curve - 880 times of minimum light

We will now apply some of the above theory to the well studied W UMa contact binary VW Cephei. 880 times of minimum light have been obtained for this system as of the summer of 1997, starting in the 1930's. When these timings are plotted using the ephemeris of Kwee (1966), we see an obvious downward-facing parabola as the first order effect.



When a parabolic least squares is applied to these residuals, we find the following solution to the quadratic equation $O - C = aE^2 + bE + c$:

c	$=$	$-3.7923618999e-3$
b	$=$	$3.0253993533e-8$
a	$=$	$-8.0584374775e-11$
r^2	$=$	0.9820708642 (correlation coefficient)

Remembering that the coefficient (a) of the quadratic term is $\frac{1}{2} \frac{dP}{dE}$ we find:

$$\frac{dP}{dE} = 2a = 2(-8.0594e - 11) = -1.61188e - 10 \frac{\text{days}}{\text{cycle}}$$

We normally convert this unit of days/cycle to sec/year:

$$\frac{dP}{dE} = (-1.61188e - 10 \frac{\text{days}}{\text{cycle}}) \left(\frac{1}{0.27831460} \frac{\text{cycle}}{\text{days}} \right) \left(86400 \frac{\text{sec}}{\text{day}} \right) \left(365.25 \frac{\text{days}}{\text{year}} \right) = -0.018 \frac{\text{sec}}{\text{year}}$$

So, the greatest effect on the period of VW Cephei is an apparently steady decrease (negative sign, i.e., downward-facing parabola) of 0.018 sec year⁻¹. If the parabola were facing upwards, it would indicate a constantly increasing period. This is most likely due to mass exchange between the components but it may also be an indication of angular momentum loss to the system due to magnetic braking. The theoretical amount of angular momentum loss (AML) can be estimated from the equation given by Bradstreet and Guinan (1994):

Calculating Angular Momentum Loss (AML) for VW Cephei

$$\frac{dP}{dt} \approx -1.1 \times 10^{-8} q^{-1} (1 + q)^2 (M_1 + M_2)^{-\frac{5}{3}} k^2 (M_1 R_1^4 + M_2 R_2^4) P^{-\frac{7}{3}}$$

where:

M_1	=	0.894 M_{\odot}		
M_2	=	0.247 M_{\odot}		
q	=	mass ratio	=	$M_2/M_1 =$ 0.272
R_1	=	0.93 R_{\odot}		
R_2	=	0.50 R_{\odot}		
k^2	=	gyration constant	=	0.10
P	=	orbital period	=	0.2783 days

$$\left(\frac{dP}{dt} \right)_{\text{theoretical AML}} = -0.006 \text{ sec yr}^{-1}$$

$$\left(\frac{dP}{dt} \right)_{\text{observed}} = -0.018 \text{ sec yr}^{-1}$$

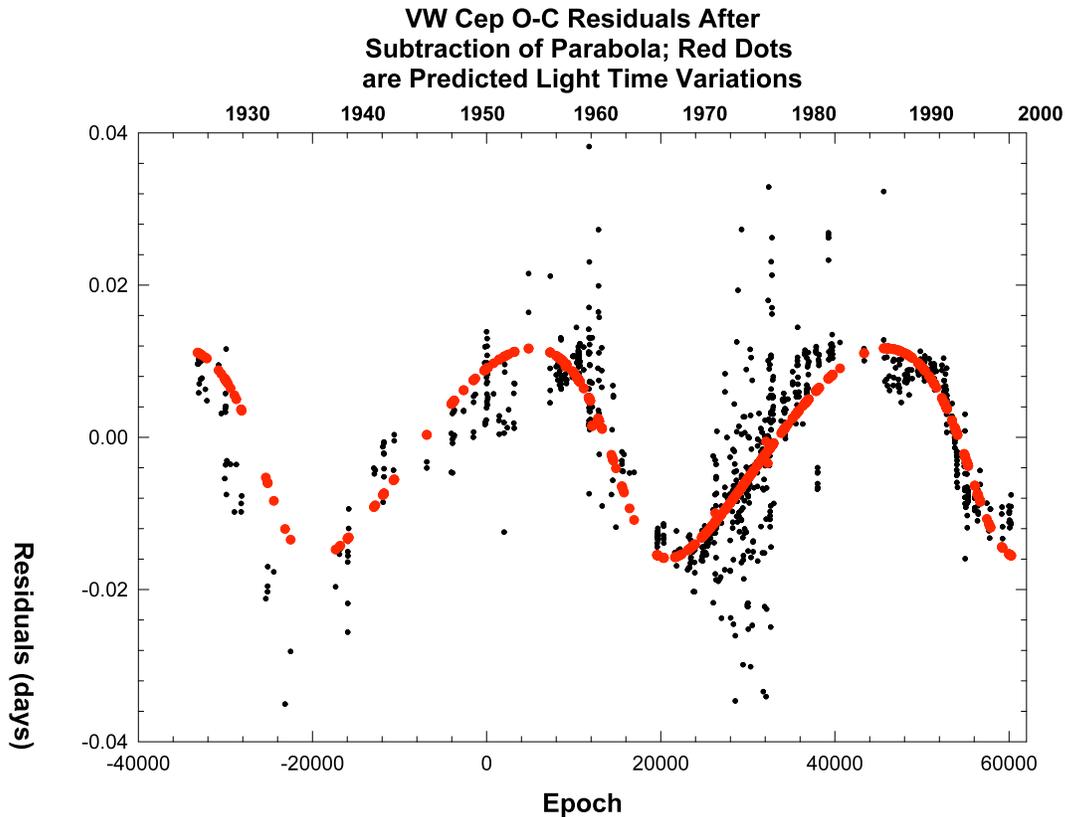
The factor of three in the observed value compared to the theoretical one may reflect some uncertainty in theory, but it more likely indicates that there is an additional evolutionary effect (like mass exchange) taking place along with the AML.

Additional Effects Visible in VW Cephei's O-C Diagram

It is obvious upon inspection of the O-C diagram for VW Cephei that there is a systematic deviation from the parabolic fit. In fact, VW Cephei has a third component star which orbits with it around a common barycenter in about 30.9 years. This tertiary body has been known for quite some time from astrometry and its effect on the O-C diagram is quite striking. We have bumped into light time effects when we discussed the

Earth's motion. This 30.9 year light time effect is due to the motion of VW Cephei about its barycenter with this third star. As this contact binary and single star orbit each other, it brings VW Cep sometimes closer and sometimes further away from us with this 30.9 year period. Thus the timings of minimum light will oscillate because sometimes VW Cep is closer to us and sometimes it is further from us.

Using previous workers' (Hershey 1975; Heintz 1993) astrometric solutions to the third body's orbit, we can derive our own solution to fit the 30.9 year oscillations based upon orbital mechanics and spherical trigonometry which exactly describe the motion of the third body. A non-linear least squares based upon the Marquardt-Levenberg algorithm (see Numerical Recipes) was developed along with a grid-searching algorithm to find the best fit curve based upon the orbital mechanics to the residuals. The curve obtained is shown below plotted against the residuals.



The solutions for the third body along with those found by Hershey and Heintz are given in the following table:

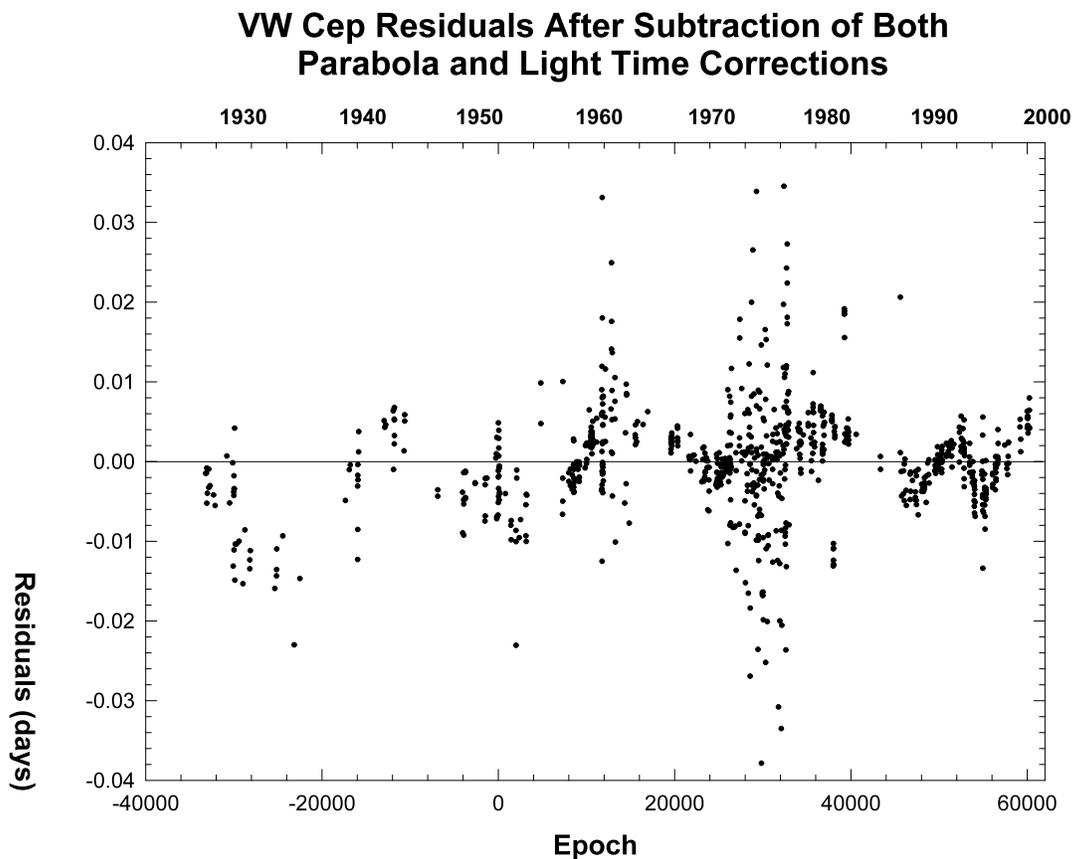
Solutions to the 3rd body orbit of VW Cep

	Hershey (1975)	Heintz (1993)	This Paper
semimajor axis a	12.4 AU	11.8 AU	11.8 AU (assumed)
period P	30.45 yr	29.0 yr	30.9 yr
periastron passage T	1966.48	1966.0	1964.0

eccentricity e	0.595	0.65	0.41
inclination i	29.2°	21.0°	47.3°
argument of periastron ω	255.5°	267.0°	205.5°
longitude of ascending node Ω	0.9°	340°	340°
mass of 3 rd body M_3	0.58 M_{\odot}	0.6 M_{\odot}	0.58 M_{\odot}

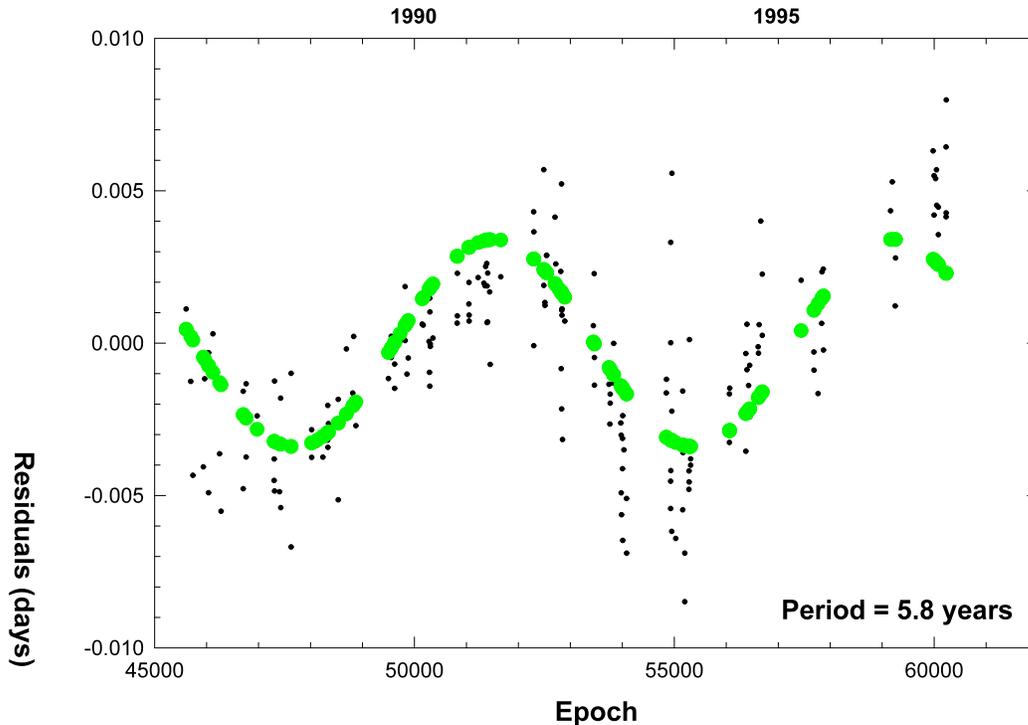
The semimajor axis for VW Cep itself is computed to be 3.50 AU.

If the third body were the only perturbation besides the overall period decrease, then after subtracting the light time correction from the residuals we should see only random scatter about the zero level. However, when we subtract the light time contribution we have the following:



There is obviously some type of non-random, periodic fluctuation of these residuals about the zero level, especially evident in the last 10 years of data. Concentrating just on these particular residuals (1986 - 1997) we fit a sinusoid to measure the approximate period of the variation. Least squares indicated a period of 5.8 years with an amplitude of 0.0033 days. This data and the sinusoidal fit are shown in the following graph. Note that the last season's residuals are not decreasing in amplitude, so this sinusoid is just a very rough approximation to what is actually happening.

VW Cep Sinusoidal Fit to the Most Recent Residuals After Parabolic and Light Time Corrections Were Applied



The following is the abstract for the poster delivered in August 1997 in Japan at the International Astronomical Union (IAU) General Assembly:

VW Cep is one of the brightest and longest observed short-period ($P= 6.67$ hours) W UMa type binaries. It consists of G5V and K0V components 11% overcontact with respect to their Roche surfaces. We investigated complex period changes based upon 880 eclipse timings from the past 70 years, including 12 obtained in the Summer of 1997 at Eastern College Observatory. In addition to the well-known 30.9 year light time effect due to the presence of a third star in the system, we find evidence for a long term decrease in the orbital period of $dP/dt= -0.018$ sec/yr. This decrease in period could arise from angular momentum loss from the binary and/or mass exchange between components. We also note a possible abrupt small change in period which took place in 1935. However, the dominant change in O-C's over the past 70 years seems to imply a rather constant

change in period because it can be fit reasonably well with a quadratic equation (*i.e.*, parabolic fit):

$$O-C = c + bE + aE^2$$

in which the coefficient $a = \frac{1}{2} \frac{dP}{dE}$. No cubic term was needed for the fit which indicates that $\frac{d^2 P}{dE^2}$ is zero or very small.

From these timings we have also refined the properties of the tertiary component and re-determined its mass and orbital parameters. We subtracted out the best fit parabola which then presumably left us with only the 3rd body light-time effects. A non-linear least squares search was then used on these residuals to arrive at the best fit orbital elements. After determining the best fit to these residuals, the light-time corrections were subtracted, leaving us small systematic deviations in the residual O-C's. These second order variations appear to be dynamical in nature and not due to the presence of a fourth body since the period of the variation has not remained constant over the 70 year observation interval. The fluctuations over the last 15 years have a period of approximately 5.8 years.

Thus the Keplerian period of the binary is presently decreasing at a fairly constant rate of $dp/dt = -0.018$ sec/yr, as well as oscillating with a ≈ 5.8 year period with a semi-amplitude of $\approx \pm 0.004$ days. The apparent scatter of the photoelectric timings taken in the same observing season most likely results from starspot activity which slightly offsets the times of primary and secondary minimum light. The time-scale of the lower amplitude residuals (5.8 years) is similar to the time-scales (5-8 years) indicated by changes in the asymmetries of the light curves and possible cyclic changes in the system's luminosity. The changes in the light curve and luminosity of the system most likely arise from growth and decay of starspots mainly on the more massive star of the binary. The starspots are believed to be of magnetic origin similar to those found in the chromospherically active RS CVn stars. If this is the case, then the 5.8 year cycle found in the O-C residuals could be in response to a magnetic activity cycle operating in the system.

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